CSE 595 Independent Study

Graph Theory

Week 2

California State University - San Bernardino

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Problem 19 (Bipartite Graphs)



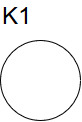
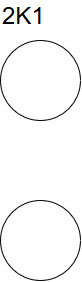
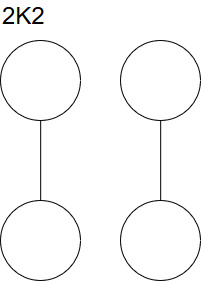
In a bipartite graph, all nodes in *U* must be connected to nodes in *W*. Therefore, there must be 60 edges from *U* to *W*, by the fact that *U* has 10 nodes each with 6 degrees. Counting the amount of edges given in *W*, it is known that there are edges from the four vertices of degree 2, and three vertices of degree 4. The remaining number of vertices is determined by their degree, which is 8. Since edges remain, nodes of degree 8 must exists in *W*.

By definition of a bipartite graph, all nodes of *G* are separated into the sets *U* and *W*. Therefore, we can say

Problem 23 (Operations on Graphs)

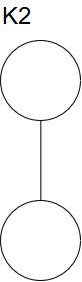


Any graph with a partite set with 3 or more nodes will have a complement with a triangle, and thus in a bipartite setting, both partite sets may not have more than 3 nodes. Therefore, size of *G* will be 4 or less.

The solution are then the following graphs,

and 2 are self-complementary because they are isomorphic to its complement.

has a compliment of



Problem 25 (Operations on Graphs)



By Theorem 1.7 in Chartrand [1],

For integers *r* and *n*, there exists an *r­*-regular graph of order *n* if and only if and *r* and *n* are not both odd.

Therefore, we know that solutions exist for both problems.

1. It is easier to look at the compliment of graph which will be denoted by . Thus, by

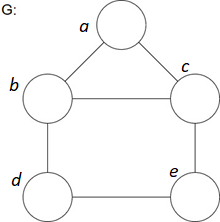
definition [1] the order and size of is determined by . Since is of order 7 and each vertex has degree of 6, then size m of is , then is of order 7 and size . We can conclude from the order and size that must be a cycle. The only two graphs that may exist are and because of the fact cycles must have order

1. Once again, it is better to look at the compliment of *G*. The order of *G* is 9 and each vertex has degree 6, then size m of *G* is =, then is of order 9 and size . must be constructed from cycles, so the possible cycles are and .

Problem 35 (Degree Sequences)



We may begin by labeling the nodes in *G* in order to transform using 2-switches to transform into *H*.

Thus, we may delete *bc* and *de* from *G* , and add *cd* and *be* In order to create the graph *H.*

Problem 37 (Degree Sequences)



1. Is not graphical. If graph *G* has 5 vertices, two of which are degree 4, then the other vertices must have degree .
2. Is graphical and can be constructed using bipartite graphs of .
3. A close up of a logo

   Description automatically generatedIs graphical and is constructed below.
4. Is not graphical. Using Theorem 1.12 (Havel-Hakimi Theorem) [1] we may apply the algorithm until either a sequence of 0’s is reached, which implies the sequence is graphical, or until a negative number is reached, which implies a nongraphical sequence.

A negative number is reached, and therefore the sequence is nongraphical.

1. A picture containing sky

   Description automatically generatedIs graphical and is constructed below.

Works cited

“Introduction.” *Graphs & Digraphs*, by Gary Chartrand et al., CRC Press, 2016, pp. 13–24.